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Letter to the Editor

Comment on two-port network analysis and modeling of a balanced armature receiver

In *Two-Port Network Analysis and Modeling of a Balanced Armature Receiver* by Kim and Allen (Hearing Research, 2013, 301:156–67), the authors published a method for obtaining Hunt parameters (T_a, Z_a and Z_e) of balanced armature receivers based on input impedance measurements with a minimum of three different acoustical loads. While the article is well-explained and thorough, some of the equations that were published in Appendix A of that article did not yield the expected results. In this letter, the development of expressions for Z_a, T_a and Z_e is detailed following the same logic as the original authors (Kim and Allen, 2013). Resulting mathematical expressions are then presented, in order to help other researchers trying to reproduce the work of the original authors.

1. Output acoustic impedance (Z_a)

As explained by Kim and Allen (2013), three measurements of the input impedance of the loudspeaker under test ($Z_{in|A}$, $Z_{in|B}$ and $Z_{in|C}$) can be represented mathematically using Hunt parameters of that loudspeaker and the three respective acoustical loads ($Z_{L|A}$, $Z_{L|B}$ and $Z_{L|C}$) that have been presented to the loudspeaker, as shown in Eqs. (1)–(3). Since the loudspeaker's input impedance can easily be measured and the acoustical loads can be chosen to have impedances that are simple to model using acoustic theory, those variables can be determined and used to obtain Hunt parameters of the loudspeaker.

$$Z_{in|A} = \frac{T_a^2}{Z_a + Z_{L|A}} + Z_e \tag{1}$$

$$Z_{in|B} = \frac{T_a^2}{Z_a + Z_{L|B}} + Z_e \tag{2}$$

$$Z_{in|C} = \frac{T_a^2}{Z_a + Z_{L|C}} + Z_e \tag{3}$$

Subtracting Eq. (1) from Eq. (3) yields Eq. (4) and subtracting Eq. (3) from Eq. (2) yields Eq. (5), after which Z_e can be eliminated.

$$Z_{in|C} - Z_{in|A} = T_a^2\left(\frac{1}{Z_a + Z_{L|C}} - \frac{1}{Z_a + Z_{L|A}}\right) \tag{4}$$

$$Z_{in|B} - Z_{in|C} = T_a^2\left(\frac{1}{Z_a + Z_{L|B}} - \frac{1}{Z_a + Z_{L|C}}\right) \tag{5}$$

Dividing Eq. (4) by Eq. (5) eliminates T_a, as shown in Eq. (6).

$$\frac{Z_{in|C} - Z_{in|A}}{Z_{in|B} - Z_{in|C}} = \frac{\left(\frac{1}{Z_a+Z_{L|C}} - \frac{1}{Z_a+Z_{L|A}}\right)}{\left(\frac{1}{Z_a+Z_{L|B}} - \frac{1}{Z_a+Z_{L|C}}\right)} \tag{6}$$

Eq. (6) is then expanded and reduced in Eq. (7) and Eq. (8).

$$\frac{Z_{in|C} - Z_{in|A}}{Z_{in|B} - Z_{in|C}} = \frac{\frac{Z_a+Z_{L|A}-Z_a-Z_{L|C}}{(Z_a+Z_{L|C})(Z_a+Z_{L|A})}}{\frac{Z_a+Z_{L|C}-Z_a-Z_{L|B}}{(Z_a+Z_{L|B})(Z_a+Z_{L|C})}} = \frac{\frac{Z_{L|A}-Z_{L|C}}{(Z_a+Z_{L|C})(Z_a+Z_{L|A})}}{\frac{Z_{L|C}-Z_{L|B}}{(Z_a+Z_{L|B})(Z_a+Z_{L|C})}} \tag{7}$$

$$\frac{Z_{in|C} - Z_{in|A}}{Z_{in|B} - Z_{in|C}} = \frac{\left(Z_{L|A} - Z_{L|C}\right)\left(Z_a + Z_{L|B}\right)\left(Z_a + Z_{L|C}\right)}{\left(Z_a + Z_{L|C}\right)\left(Z_a + Z_{L|A}\right)\left(Z_{L|C} - Z_{L|B}\right)} = \frac{\left(Z_{L|A} - Z_{L|C}\right)\left(Z_a + Z_{L|B}\right)}{\left(Z_a + Z_{L|A}\right)\left(Z_{L|C} - Z_{L|B}\right)} \tag{8}$$

Reorganizing Eq. (8) leads to Eq. (9).

$$Z_a + Z_{L|A} = \frac{\left(Z_{L|A} - Z_{L|C}\right)\left(Z_{in|B} - Z_{in|C}\right)}{\left(Z_{in|C} - Z_{in|A}\right)\left(Z_{L|C} - Z_{L|B}\right)}\left(Z_a + Z_{L|B}\right) \tag{9}$$

Defining parameter K in Eq. (10) allows Eq. (9) to be rewritten into Eq. (11).

$$K = \frac{\left(Z_{L|A} - Z_{L|C}\right)\left(Z_{in|B} - Z_{in|C}\right)}{\left(Z_{in|C} - Z_{in|A}\right)\left(Z_{L|C} - Z_{L|B}\right)} \tag{10}$$

$$Z_a + Z_{L|A} = K\left(Z_a + Z_{L|B}\right) \tag{11}$$

It is then possible to isolate Z_a, leading to Eq. (12). While this equation could be expanded and rearranged, it is not mathematically equivalent to the expression of Z_a given in Eq. (A.3) by Kim and Allen (2013).

$$Z_a = \frac{KZ_{L|B} - Z_{L|A}}{(1-K)} \tag{12}$$

2. Acoustic transduction impedance (T_a)

With Z_a solved, it is possible to solve for T_a by re-arranging Eq.

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(4) into Eq. (13–15). It can be seen that Eq. (15) differs from Eq. (A.4) of Kim and Allen (2013) by the sign "±". This sign can be adjusted according to the polarity convention of the loudspeaker under test and affects the phase of the loudspeaker response.

$$Z_{in|C} - Z_{in|A} = T_a^2\left(\frac{1}{Z_a + Z_{L|C}} - \frac{1}{Z_a + Z_{L|A}}\right) = T_a^2\frac{\left(Z_{L|A} - Z_{L|C}\right)}{\left(Z_a + Z_{L|C}\right)\left(Z_a + Z_{L|A}\right)} \tag{13}$$

$$T_a^2 = \frac{\left(Z_{in|C} - Z_{in|A}\right)\left(Z_a + Z_{L|C}\right)\left(Z_a + Z_{L|A}\right)}{\left(Z_{L|A} - Z_{L|C}\right)} \tag{14}$$

$$T_a = \pm\sqrt{\frac{\left(Z_{in|C} - Z_{in|A}\right)\left(Z_a + Z_{L|C}\right)\left(Z_a + Z_{L|A}\right)}{\left(Z_{L|A} - Z_{L|C}\right)}} \tag{15}$$

3. Electrical input impedance (Z_e)

With Z_a and T_a known, Z_e can be solved by using Eq. (1), yielding Eq. (16). This equation also differs from Eq. (A.5) given by Kim and Allen (2013).

$$Z_e = Z_{in|A} - \frac{T_a^2}{Z_a + Z_{L|A}} \tag{16}$$

Reference

Kim, Noori, Allen, Jont B., 2013. Two-port network analysis and modeling of a balanced armature receiver. Hear. Res. 301, 156–167.

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